$$\frac{dE_k}{dt} = 4\pi r_1^2 \rho_0 D^3 \left[\frac{3(1 - a^2/D^2)}{3+m} - \frac{(1 - a^4/D^4)}{2} \right], \qquad (12)$$

where $E_k = 4\pi \int_0^{r_1} \frac{\rho v^2}{2} r^2 dr$ is the kinetic energy of the moving gas.

We note that when $M_0 \to 0$, systems (7)-(12) convert to the corresponding equations in [1]. From the conditions (6) when m = 0 and $\rho_0/\rho_1 \ll 1$, it follows that the initial dispersion of the mass released takes place under the self-similar conditions considered in [3]. When m = 0, the distributions (6) are satisfied by the equation of continuity (1) for any function $r_1(t)$.

As initial values for the problem of numerical computation when t = 0, we take $r_{T_0} = 30$ m, $r_1 = 0$, $\alpha = 0.5$, m = 0, $\rho_0 = 1.29$ kg/m³. Consequently, for t = 0, $\rho_1 r_1^3 = 3M_0/4\pi$.

Figure 1 shows the relations between the radii of the thermal and shock waves and the time. Curves 1-3 correspond to the values $M_0 = 0$ (self-similar solution [4]), 0.1, and 10 t. It can be seen that the mass released depends significantly on the conditions of propagation of the thermal and shock waves.

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DEVELOPMENT OF DYNAMIC PERTURBATIONS IN INITIAL STAGE OF A POINT EXPLOSION IN A THERMALLY CONDUCTING GAS

V. P. Shidlovskii

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The one-dimensional perturbations originating in a cold homogeneous gas ($T_1 = 0$, ρ_1 = const) by the instantaneous release of finite energy at the origin of the coordinates are considered. The starting equations are compiled for a gas in which the heat-transfer mechanism is simulated by a nonlinear thermal conductivity with coefficient $\lambda \sim T^n$. Transformation of the equations to the dimensionless form by the introduction of "natural" variable allows the simplest path for investigating the process as a whole to be shown by means of the method of perturbations. The initial approximation corresponds to the well-known solution for a thermal wave [1], while subsequent approximations describe the joint development of both thermal and dynamic perturbations. An investigation of the properties of the solutions and an example of the calculation of the first two approximations (without taking account of the starting approximation) for the case of a point spherical explosion with n = 5 gives a representation of the formation of the shock wave.

When studying an explosion in a gas, it is of great importance to take into account the actual heat-transfer processes. This is especially important in the very first stage of the explosion or, as observations and theoretical investigations [2] show, the thermal wave originates during the explosion even before the appearance of the dynamic nature of the phenomenon. The heat-transfer mechanism, in this case, is due mainly to the effect of radiation, but if we neglect the pressure and the radiation energy, then a completely acceptable

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description of this mechanism is provided by the nonlinear thermal-conductivity model. The investigation of the thermal waves originating in a cold gas in the case of nonlinear thermal conductivity is given in [1]. A more general approach is proposed below, which takes into account not only thermal but also dynamic processes.

§1. The system of equations which is suitable for describing the one-dimensional irregular processes in a thermally conducting gas is written in the form

$$\frac{\partial \rho}{\partial t} + v\frac{\partial \rho}{\partial r} + \rho\left(\frac{\partial v}{\partial r} + \frac{\nu-1}{r}v\right) = 0, \quad \rho\left(\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r}\right) + \frac{\partial p}{\partial r} = 0,$$
$$\rho\left(\frac{\partial e}{\partial t} + v\frac{\partial e}{\partial r}\right) + p\left(\frac{\partial v}{\partial r} + \frac{\nu-1}{r}v\right) = r^{1-\nu}\frac{\partial}{\partial r}\left[r^{\nu-1}\lambda\frac{\partial(e/c_V)}{\partial r}\right],$$
$$p = (\varkappa - 1)\rho e, \tag{1.1}$$

where t is the time; r is a coordinate; ρ, v, p, and e are the density, velocity, pressure, and internal energy of the gas, respectively ($e = c_V T$); $\varkappa = c_p/c_V$; and ν is the parameter of symmetry (ν = 1, 2, 3).

The thermal-conductivity coefficient of the gas λ is assumed to be expressed by the formula

$$\lambda = c_V A e^n \quad (A,\ n = \text{const}).$$

If the initial temperature of the gas can be assumed equal to zero, then for any value of $t > 0$ the region of the perturbed state will be bounded by the perturbation front $r = r_f(t)$. In the case of a point explosion, when for $t = 0$ at the origin of the coordinates a finite energy E is released, the initial and boundary conditions are formulated in the form

$$\begin{aligned} &p = e = v = 0, && \text{for } t = 0,\\ &\rho = \rho_1,\ \lambda\partial e/\partial r = 0 && \text{for } t > 0,\ r = r_f,\\ &v = \lambda\partial e/\partial r \neq 0 && \text{for } t > 0,\ r = 0, \end{aligned} \tag{1.2}$$

where the initial density ρ_1 will be assumed constant. In addition to this, the integral condition of constancy of the total energy of the perturbed gas volume must be satisfied

$$\xi_\nu \int_0^{r_f} \rho\,(e + v^2/2)\,r^{\nu-1}dr = E, \quad \xi_\nu = 2(\nu-1)\pi + (\nu-2)(\nu-3)/2, \ E = \text{const}. \tag{1.3}$$

Equations (1.1) can be conveniently converted to dimensionless form by introducing the new arguments

$$\eta = \frac{r}{r_f}, \qquad \chi = \frac{A}{(\varkappa-1)^n}\frac{U^{2n-1}}{\rho_1 r_f}, \tag{1.4}$$

where $U = dr_f/dt$ is the propagation velocity of the perturbation front. These arguments can be called natural as the first of them represents the normalized space coordinate and the second one a natural dimensionless combination depending on the time and which does not contain the coordinate r.

Transformation of the required variables is also effected by the introduction of "natural" scales

$$v = UV(\eta,\chi),\ \rho = \rho_1 R(\eta,\chi),\ p = \rho_1 U^2 P(\eta,\chi),$$
$$e = (\varkappa-1)^{-1}U^2 N(\eta,\chi),\ \lambda = c_V\rho_1 r_f U\chi N^n(\eta,\chi).$$

After conversion to the new variable, Eqs. (1.1) assume the form

$$(V-\eta)\frac{\partial R}{\partial \eta} + K\chi\frac{\partial R}{\partial \chi} + R\frac{\partial V}{\partial \eta} + \frac{\nu-1}{\eta}RV = 0,$$
$$R\left[ZV + (V-\eta)\frac{\partial V}{\partial \eta} + K\chi\frac{\partial V}{\partial \chi}\right] + \frac{\partial P}{\partial \eta} = 0,$$
$$R\left[2ZN + (V-\eta)\frac{\partial N}{\partial \eta} + K\chi\frac{\partial N}{\partial \chi} + (\varkappa-1)N\left(\frac{\partial V}{\partial \eta} + \frac{\nu-1}{\eta}V\right)\right] =$$
$$= \chi\eta^{1-\nu}\frac{\partial}{\partial \eta}\left(\eta^{\nu-1}N^n\frac{\partial N}{\partial \eta}\right), \quad P = RN. \tag{1.5}$$

Here the notation

$$Z(\chi) = (dU/dt)r_f U^{-2},\ K(\chi) = (2n-1)Z(\chi) - 1. \tag{1.6}$$

is introduced.

The boundary conditions obtained from Eq. (1.2) have the form

$$R = 1,\ P = N = V = N^n\, dN/\partial\eta = 0 \ \text{ for } \ \eta = 1, \qquad V = N^n\, dN/\partial\eta = 0 \ \text{ for } \ \eta = 0. \tag{1.7}$$

Before transforming condition (1.3), we introduce the new function

$$\Psi(\chi) = \int_0^1 R\left[(\varkappa - 1)^{-1}N + V^2/2\right]\eta^{\nu-1}d\eta.$$

Taking account of the notation in Eq. (1.6), condition (1.3) can be represented in the form

$$\chi K d\Psi/d\chi + (2Z + \nu)\Psi = 0.$$

§2. Resulting from the determination (1.4) of the variable χ now replacing the time, the following can be substituted. The limit $\chi \to 0$ corresponds to the conversion of Eq. (1.5) to the equation for the adiabatic self-similar motion of a gas during a powerful explosion [3]. The physical properties of the phenomenon being considered confirm that this limit corresponds to an infinitely large time $t \to \infty$. Introducing the natural assumption that the function $\chi(t)$ is monotonic, we arrive at the conclusion that the initial stage of the process being investigated corresponds to the limit $\chi \to \infty$. We shall try to find the limiting form of Eq. (1.5) for large values of χ, assuming that it will not contain an explicit dependence on χ and in the energy equation, both the term linked with the thermal conductivity and also the term defining the change with time, must be conserved. For this purpose, we change the scales of part of the unknown variables by introducing new functions according to the formulas

$$N = B\chi^\alpha f(\eta, \chi),\ V = \chi^{\alpha/2}g(\eta, \chi),\ P = B\chi^\alpha h(\eta, \chi), \tag{2.1}$$

where B is a normalized constant [$B = O(1)$] introduced for the convenience of comparison with the solutions obtained earlier. The condition referred to above, relative to the order of magnitude of the terms of the energy equation, allows us to find

$$\alpha = -1/n. \tag{2.2}$$

Equations (1.5), taking Eqs. (2.1) and (2.2) into account, assume the form

$$\begin{gathered}
(\chi^{-1/2n}g - \eta)\frac{\partial R}{\partial \eta} + K\chi\frac{\partial R}{\partial \chi} + \chi^{-1/2n}R\left(\frac{\partial g}{\partial \eta} + \frac{\nu - 1}{\eta}g\right) = 0, \\
R\left[Zg + (\chi^{-1/2n}g - \eta)\frac{\partial g}{\partial \eta} - \frac{1}{2n}Kg + K\chi\frac{\partial g}{\partial \chi}\right] + \chi^{-1/2n}B\frac{\partial h}{\partial \eta} = 0, \\
R\left[2Zf + (\chi^{-1/2n}g - \eta)\frac{\partial f}{\partial \eta} - \frac{1}{n}Kf + K\chi\frac{\partial f}{\partial \chi} + \right. \\
\left. + (\varkappa - 1)\chi^{-1/2n}f\left(\frac{\partial g}{\partial \eta} + \frac{\nu - 1}{\eta}g\right)\right] = \frac{B^n}{n+1}\eta^{1-\nu}\frac{\partial}{\partial \eta}\left(\eta^{\nu-1}\frac{\partial f^{n+1}}{\partial \eta}\right), \quad h = Rf.
\end{gathered} \tag{2.3}$$

System (2.3) has a form which is very suitable not only for obtaining the limiting form of the equations for $\chi \to \infty$, but also for constructing the solutions which correspond to the finite form, although for quite large values of χ. A number of terms occurring in Eq. (2.3) contain the factor $\chi^{-(1/2n)}$ and it will be natural to try to find these solutions in the form of series

$$F(\eta, \chi) = F_0(\eta) + \sum_i \chi^{-i/2n}F_i(\eta), \tag{2.4}$$

where F is any of the functions f, g, h, or R. Similar series can also be used for representing the functions $Z(\chi)$ and $K(\chi)$ for large values of χ.

If we substitute function (2.4) in Eq. (2.3) and convert to the limit $\chi \to \infty$, we obtain the equations of the initial approximation, i.e., the limiting form of the starting equations corresponding to the initial stage of the explosion:

$$\frac{dR_0}{d\eta}=0,\quad R_0\left[\left(Z_0-\frac{1}{2n}K_0\right)g_0-\eta\frac{dg_0}{d\eta}\right]=0,$$

$$R_0\frac{b_\nu}{\nu n+2}\left[2\left(Z_0-\frac{1}{2n}K_0\right)f_0-\eta\frac{df_0}{d\eta}\right]=\eta^{1-\nu}\frac{d}{d\eta}\left(\eta^{\nu-1}\frac{df_0^{n+1}}{d\eta}\right).$$

We shall suppose that the constant B which occurs in formula (2.1) is equal to

$$B=\left[\frac{(\nu n+2)(n+1)}{b_\nu}\right]^{1/n},\quad b_\nu^{\frac{\nu n+2}{2n}}=2\left[\frac{2(\nu n+2)(n+1)}{n}\right]^{1/n}\Gamma\left(\frac{\nu}{2}+\right.$$

$$\left.+\frac{n+1}{n}\right)\left[\Gamma\left(\frac{\nu}{2}\right)\Gamma\left(\frac{n+1}{n}\right)\right]^{-1}.$$

Taking into account the boundary conditions $R_0(1) = 1$, $g_0(1) = 0$, obtained from Eq. (1.7), we obtain $R_0 \equiv 1$ and $g_0 \equiv 0$.

Let us return to condition (1.8). In accordance with Eq. (2.1), we assume

$$\Psi(\chi)=\chi^{-1/n}\left(a_0+\sum_i\chi^{-i/2n}a_i\right), \tag{2.5}$$

where a_0 and a_i are certain numbers. In the initial approximation we obtain

$$-K_0/n+2Z_0+\nu=0,$$

whence, taking account of Eq. (1.6), it follows that

$$Z_0=-(\nu n+1),\ K_0=-n[\nu(2n-1)+2],$$

and, in accordance with the definition of Z, the law of motion of the perturbation front is also established

$$r_f=\gamma t^{1/(\nu n+2)},$$

where the constant factor γ can be expressed in terms of the defining parameters of the problem, including the energy E.

Thus, the construction of the initial approximation reduces to the solution of the single equation

$$\frac{d}{d\eta}\left(\eta^{\nu-1}\frac{df_0^{n+1}}{d\eta}\right)=-\frac{b_\nu}{\nu n+2}\frac{d}{d\eta}(\eta^\nu f_0) \tag{2.6}$$

with the boundary conditions

$$f_0(1)=(df_0^{n+1}/d\eta)_{\eta=1}=0.$$

Equation (2.6) coincides in accuracy with the equation of the thermal wave problem [4], which was obtained without association with the dynamic problem. The solution of Eq. (2.6) has the form

$$f_0=D_f(1-\eta^2)^{1/n},\quad D_f^n=\frac{nb_\nu}{2(n+1)(\nu n+2)}. \tag{2.7}$$

The graph of the function $f_0(\eta)$ is shown in Fig. 1.

§3. Let us carry out the construction of the functions of the first approximation. From the first equation of system (2.3) and from the condition $R_1(1) = 0$, $R_1 \equiv 0$ can be found. For the function f_1, an equation of the form

$$\eta^{1-\nu}\frac{d}{d\eta}\left[\eta^{\nu-1}\frac{d}{d\eta}(f_0^nf_1)\right]=-\frac{b\nu}{(n+1)(\nu n+2)}\left[\eta^{1-\nu}\frac{d}{d\eta}(\eta^\nu f_1)-\frac{Z_1}{n}f_0\right]. \tag{3.1}$$

is obtained.

The parameter Z_1 is unknown beforehand and therefore we represent

$$f_1=f_{11}+Z_1f_{12},$$

where f_{11} is the solution of the homogeneous equation obtained from Eq. (3.1). Taking into account Eq. (2.7), we obtain $f_{11} = C(1 - \eta^2)^{(1-n)/n}$. If we take into consideration that we are interested in values of $n > 1$, the fulfillment of the boundary condition $f_{11}(1) = 0$ proves to be possible only if $C = 0$, whence $f_{11} \equiv 0$.

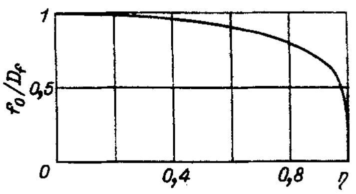

Fig. 1

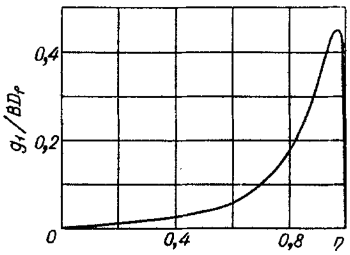

Fig. 2

After this, the parameter a_1 [see Eq. (2.5)] can be represented in the form

$$a_1 = Z_1 B(\varkappa - 1)^{-1} \int_0^1 f_{12}\eta^{\nu-1} d\eta = Z_1 k_1.$$

Equation (1.8) in the approximation being considered, taking account of Eq. (2.3), has the form

$$Z_1(-K_0 k_1/2 + a_0) = 0. \tag{3.2}$$

In the general case, the expression in the brackets on the left-hand side of Eq. (3.2) is nonzero, whence it follows that $Z_1 = 0$ and, thus, we have finally $f_1 \equiv 0$.

When calculating the results of the zero approximation, the function $g_1(\eta)$ determines the principal term of the expression for the gas velocity. In order to determine g_1, we obtain the equation (the primes denote differentiation with respect to η).

$$\eta g_1' - [\nu(n-1)+1]\, g_1 = B f_0'. \tag{3.3}$$

After integration of Eq. (3.3), taking account of the boundary condition, we obtain

$$g_1 = B\eta^{\nu(n-1)+1} \int_1^\eta f_0' \eta^{-[\nu(n-1)+2]} d\eta. \tag{3.4}$$

Reverting to expression (2.7) and introducing the notation

$$x = (1 - \eta^2)^{1/n}, \tag{3.5}$$

we can rewrite Eq. (3.5) in some other form

$$g_1 = D_f B \eta^{\nu(n-1)+1} \int_0^x (1 - x^n)^{-\frac{1}{2}[\nu(n-1)+2]} dx. \tag{3.6}$$

If n is an odd whole number, then the square in the right-hand side of Eq. (3.6) can be calculated analytically, but in the general case one should turn to a numerical method. Certain characteristic features of the function $g_1(\eta)$ can be shown which appear for any values of $g_1(\eta)$ and for any values of $n > 1$.

At the center of the explosion, formula (3.6) gives $g_1(0) = 0$ with a finite differential quotient.

$$g_1'(0) = \frac{2}{n(n-1)\nu} BD_f .$$

In the vicinity of the front $g_1(\eta)$ behaves the same as $f_0(\eta)$, i.e., as $(1 - \eta^2)^{1/n}$.

The graph of the function $g_1(\eta)$ for $\nu = 3$ and $n = 5$ is shown in Fig. 2. Despite this function specifying a "small perturbation," in its form it is very reminiscent of the discontinuous function of adiabatic motion.

The principal term of the dynamic density perturbation is expressed by the function $R_2(\eta)$. The equation for determining this function has the form

$$\eta R_2' + n^{-1} K_0 R_2 - \eta^{1-\nu}(\eta^{\nu-1} g_1)' = 0. \tag{3.7}$$

With the condition $R_2(1) = 0$, the solution of Eq. (3.7) can be represented in the form

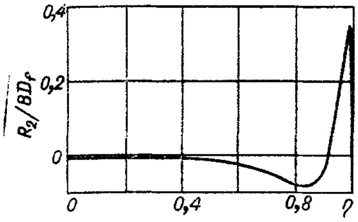

Fig. 3

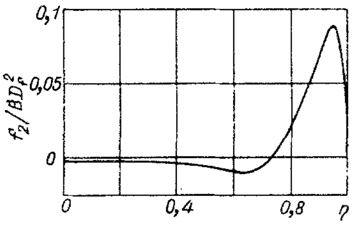

Fig. 4

$$R_2 = -\frac{\nu n}{\nu n+2}\frac{g_1}{\eta} + \frac{2(\nu n+1)}{\nu n+2} BD_f \eta^{\nu(2n-1)+2} \int_0^x (1-x^n)^{-\frac{1}{2}[\nu(2n-1)+4]} dx, \qquad (3.8)$$

where the substitution (3.5) is used again. Just as for the determination of g_1, the solution has been reduced to a single square.

Using Eq. (3.8), we can show certain properties of the function $R_2(\eta)$. Just like the function $g_1(\eta)$, at the perturbation front it becomes zero and behaves in the vicinity of this point as $(1-\eta^2)^{1/n}$. At the center, it arrives with zero derivative, assuming a finite value there:

$$R_2(0) = -2BD_f/n(n-1)[\nu(2n-1)+2].$$

The result of a numerical calculation of the function $R_2(\eta)$ for $\nu = 3$ and $n = 5$ is shown in Fig. 3; it can be seen that on a considerable part of the perturbed region the quantity R_2 varies only slightly, maintaining small negative values. The main changes occur near the front and, just as in the case of the velocity, the compression wave in the frontal zone strongly resembles the shock wave.

In explaining the tendency to the development of a dynamic temperature perturbation, the function $f_2(\eta)$ satisfying the equation

$$\begin{aligned} &B^n\eta^{1-\nu}\left[\eta^{\nu-1}\left(f_0^n f_2\right)'\right]' + \eta^{2\nu(n-1)+3}\left[\eta^{-2[\nu(n-1)+1]} f_2\right]' = \\ &= n^{-1}Z_2 f_0 - R_2\eta^{1-\nu}\left(\eta^\nu f_0\right)' + g_1 f_0' + (\varkappa-1)\eta^{1-\nu} f_0\left(\eta^{\nu-1}g_1\right)'. \end{aligned} \qquad (3.9)$$

should be determined.

The boundary conditions for $f_2(\eta)$ are obtained from Eq. (1.7) in the form

$$f_2(1) = 0, \quad (1-\eta^2) f_2'(\eta) = 0 \quad \text{for} \quad \eta = 0 \text{ and } \eta = 1.$$

Parallel with the determination of $f_2(\eta)$, the quantity Z_2 must also be found, and therefore the form

$$f_2 = f_{21} + Z_2 f_{22}$$

is used again.

Reverting to the form $\Psi(\chi)$, in accordance with Eq. (2.5), we obtain likewise

$$a_2 = a_{21} + Z_2 a_{22},$$

where

$$a_{21} = \int_0^1 \left[B(\varkappa-1)^{-1}(f_0 R_2 + f_{21}) + g_1^2/2\right]\eta^{\nu-1} d\eta;$$

$$a_{22} = B(\varkappa-1)^{-1}\int_0^1 f_{22}\eta^{\nu-1} d\eta.$$

If the functions f_{21} and f_{22} are determined, then the quantities a_{21} and a_{22} can also be found, after which, by means of Eq. (1.8), we obtain

$$Z_2 = a_{21}K_0/(a_0 - a_{22}K_0).$$

The graph of the function $f_2(\eta)$ for these same parameters $\nu = 3$ and $n = 5$ is shown in Fig. 4. The number of boundary conditions for Eqs. (3.3) and (3.9) is greater by unity than the order of the equations themselves. Fulfillment of the "superfluous" boundary conditions is ensured here because of the properties of the equations themselves; however, in principle, we are not obliged to limit the required solutions only to the class of continuous functions. From both theoretical considerations and analysis of the experimental data, it is well known (see, for example, [2]), that in the region between the center of the explosion and the perturbation front, with defined conditions, strong discontinuities can originate. Under the conditions of the example chosen and within the framework of the approximations considered here, these discontinuities are not developed.

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REPRESENTATION OF INTERACTION IN THE THEORY OF TURBULENCE

G. A. Kuz'min and A. Z. Patashinskii

UDC 532.517.4

The concept that in a turbulent flow energy exchanges only take place of pulsations of near scales is the basis of macroscopic theory of local turbulence structure. Universality and similarity of small-scale statistical pulsations are inferred from the assumption that the energy exchange is of random character. In the Eulerian equations of motion, together with the interactions which implement the energy exchange between pulsations, there are fictitious interactions related to the transfer of pulsations of a given scale l by the pulsations of scales $l' \gg l$. It was emphasized in [2, 3] that in the Eulerian description of turbulence the effect of transfer results in a strong statistical dependence of pulsations of different scales. Therefore, the universality and similarity of small-scale pulsations can be observed only in these variables in which there are no effects of pure transfer of some pulsations by the others. Qualitative considerations were therefore given in [1-3] on the need for describing small-scale pulsations in a reference system which is in motion at each point with all large-scale pulsations. It is shown in the present article that such description of small-scale pulsations can be implemented with the aid of transfer representation similar to the representation of interaction in the quantum field theory [4]. Representation of interaction is of intermediate position between the Lagrangian and Eulerian descriptions of turbulence, since a transfer of a packet as an entity can be described in variables which are Lagrangian only as regards large-scale motions. Another way of eliminating transfer interactions is based on the introduction of nonsolenoid velocity as in [5]. From the physical point of view, the method employed in this article seems to be more appropriate.

First, the case of the scalar field $\varphi(\mathbf{x}, t)$ is considered; its entire evolution in time is related to the transfer of the field φ to the velocity field $\mathbf{v}(x, t)$. The part of the field φ can be taken, for example, by the concentration of a passive admixture in a turbulent flow. The equation for φ is

$$\partial\varphi/\partial t + (\mathbf{v}\nabla)\varphi = 0. \tag{1}$$

By integrating (1) with respect to time one obtains the integral equation

$$\varphi(\mathbf{x}, t) = \varphi(\mathbf{x}, t_0) - \int_{t_0}^{t} d\tau\,(\mathbf{v}(\mathbf{x}, \tau)\nabla)\,\varphi(\mathbf{x}, \tau).$$

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